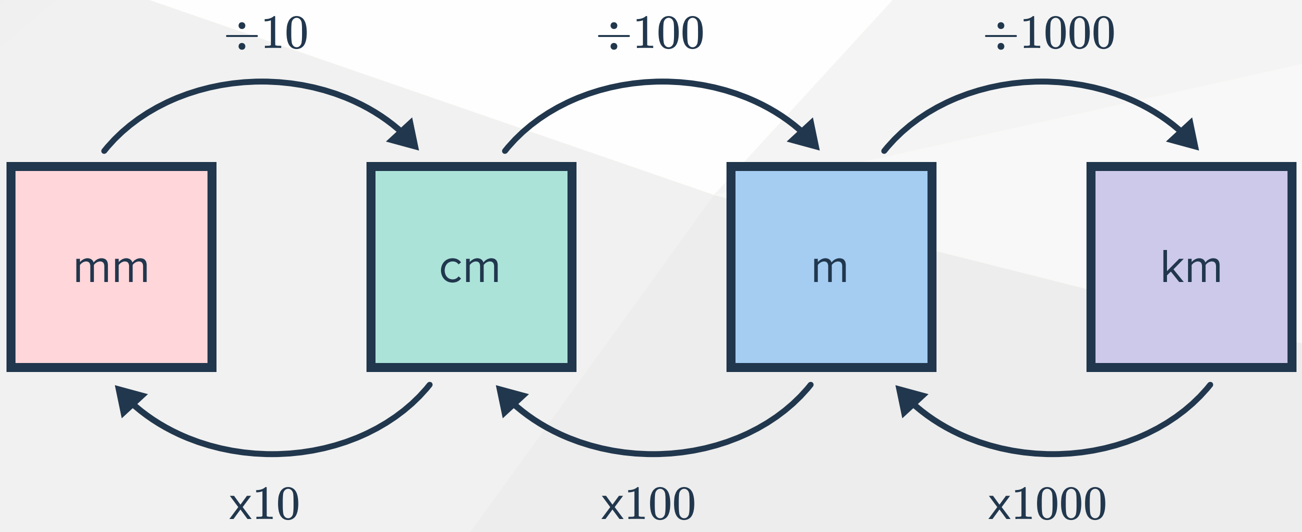
Measurement

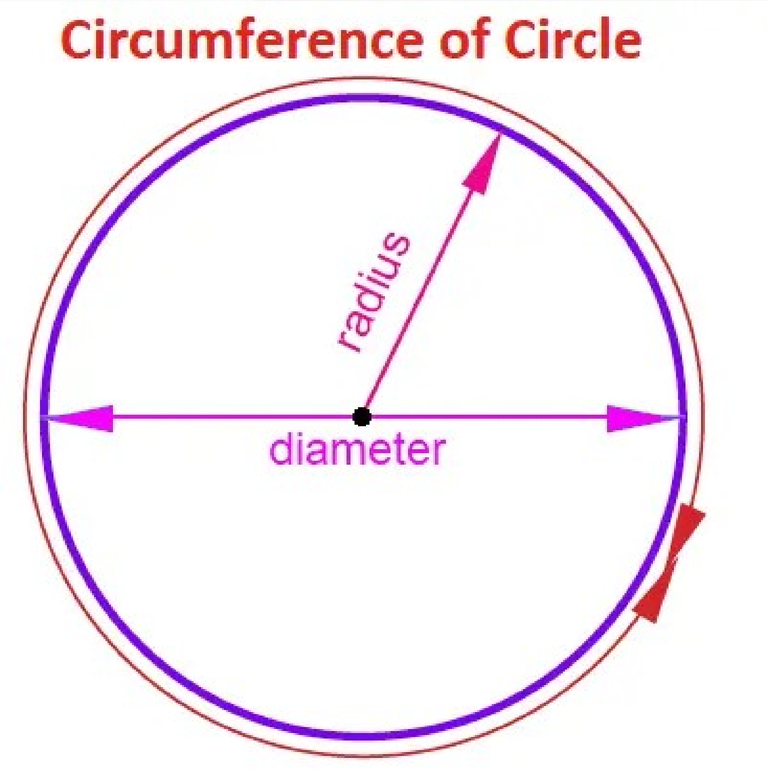
Review: Length

- What's perimeter?
  - Perimeter is the distance around the outside of a closed shape

Metric units of length



Lengths of a Circle



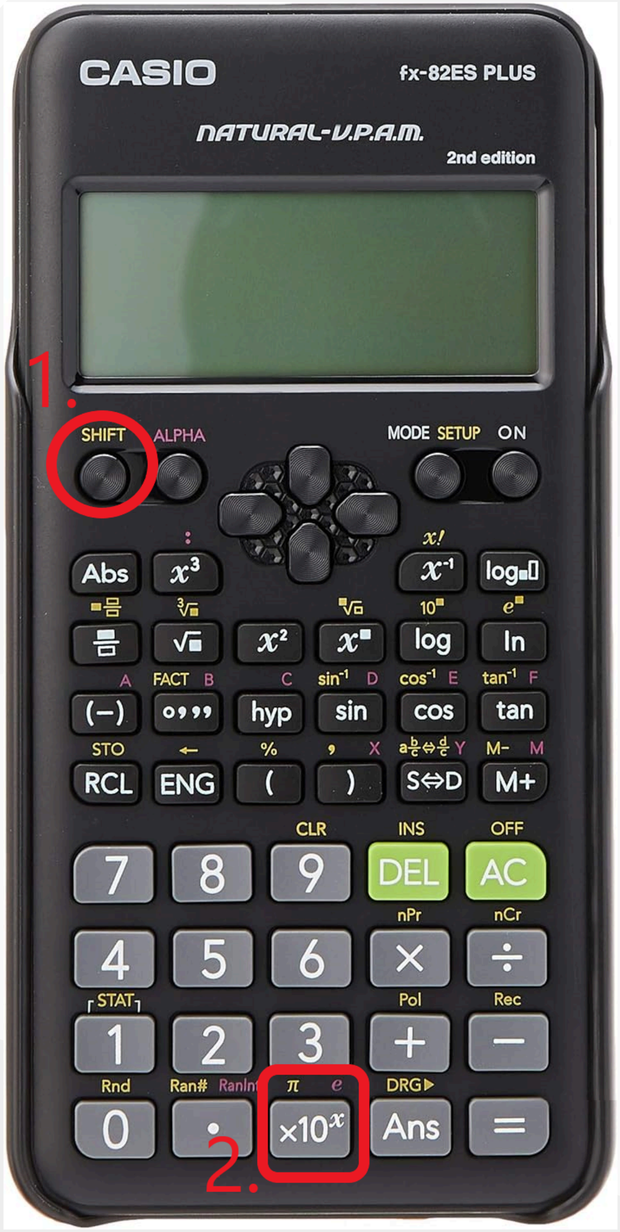
- **circumference**: perimeter of a circle
- **radius**: distance from circle's centre to its edge
- **diameter**: distance from one end of the circle to the other, through the centre

Circumference Formula

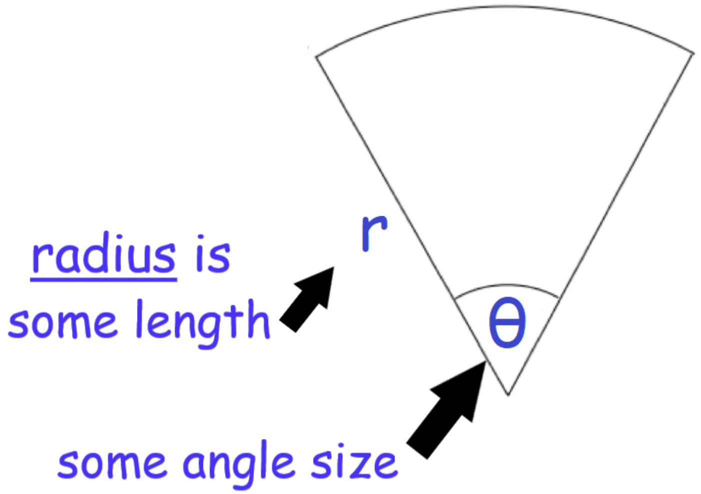
- $\boxed{\text{Circumference} = \pi d}$
- $\boxed{\text{Circumference} = 2\pi r}$
  - Where, d = diameter, r = radius (remember: diameter = $2\times$ radius) and $\pi \approx 3.14$

Remembering Pi π

- irrational number but approximately 3.14
- π is on your calculator: Let's find it
- In most calculators, it's on the *bottom* near the *right* of the numbers ↘️
- Press SHIFT then $\times 10^x$

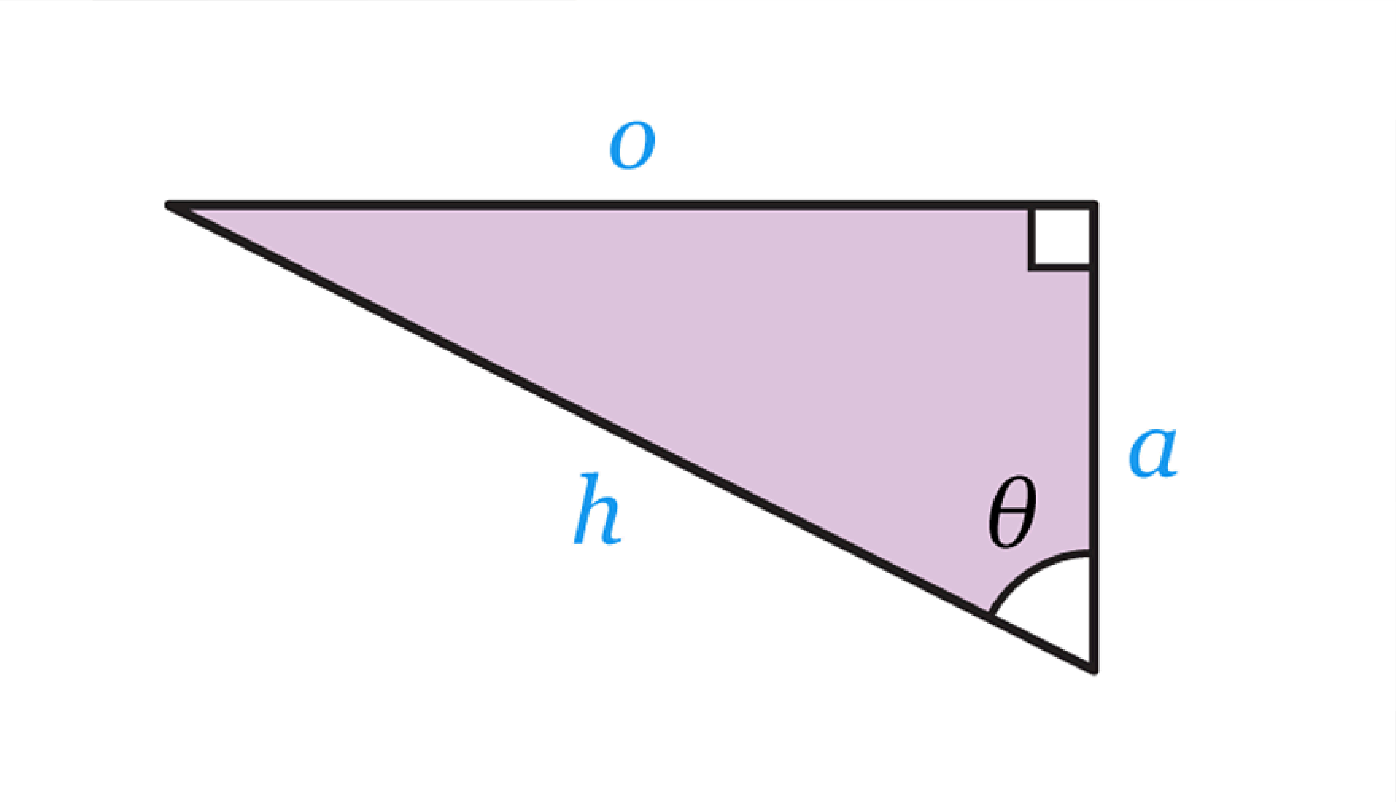


Perimeter of a Sector



- Remember: Circumference = $2\pi r$
- $\text{Arc-length} = \frac{\theta}{360^\circ} \times 2\pi r$
- Perimeter = arc-length + $2r$
- $\text{Perimeter of a sector} = 2r + \frac{\theta}{360^\circ} \times 2\pi r$
  - where r is the radius and θ is the angle of the sector
- Note: θ is not 0: it is a Greek symbol that just looks similar

What's the relationship between o, a and h?



Review: Pythagoras' theorem

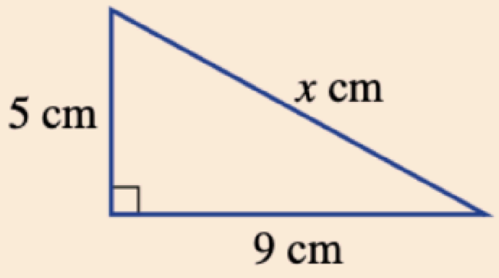
$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$

where a and b are the short sides and c is the hypotenuse

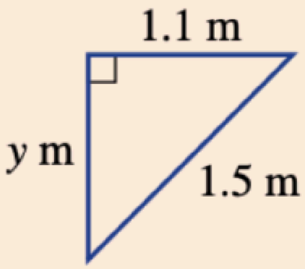
Example 13 **Finding side lengths using Pythagoras' theorem**

Find the length of the unknown side in these right-angled triangles, correct to two decimal places.

a

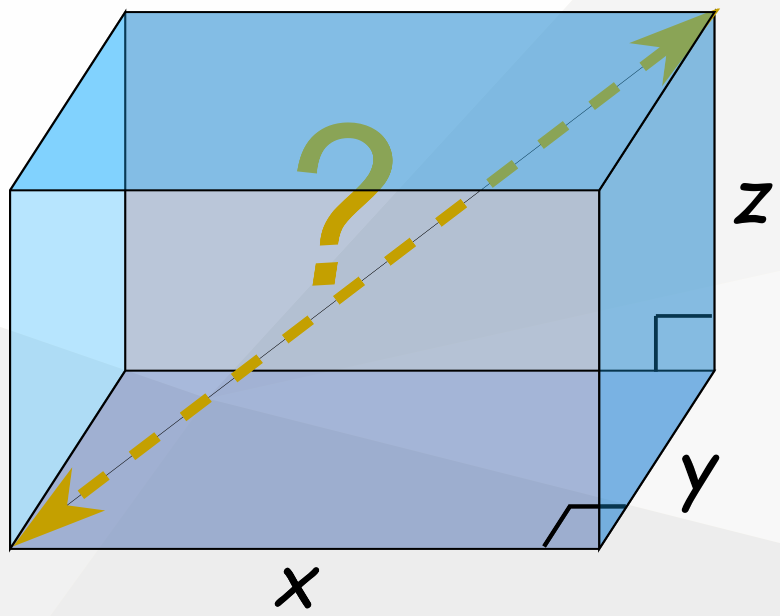


b



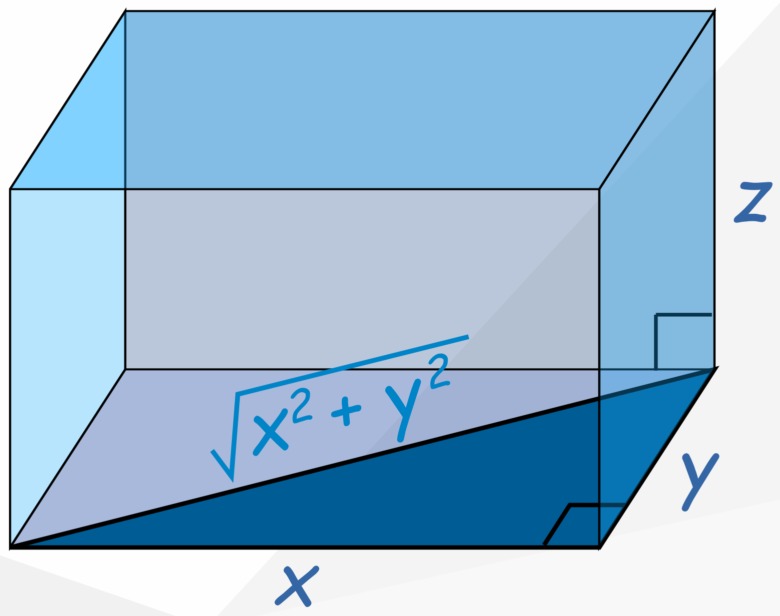
Pythagoras NOW IN 3D

How would we find the diagonal of this cuboid?



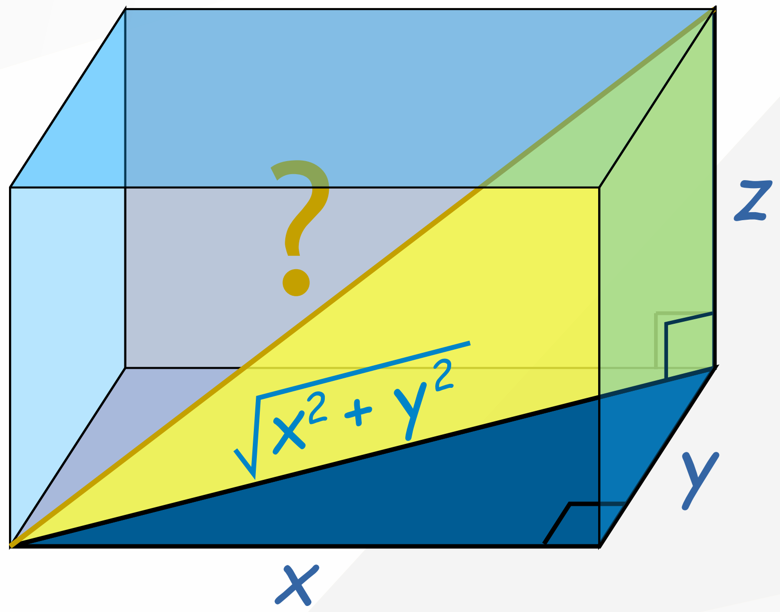
- What can we find first?

Using Pythagoras's theorem, we can find the diagonal of the bottom face



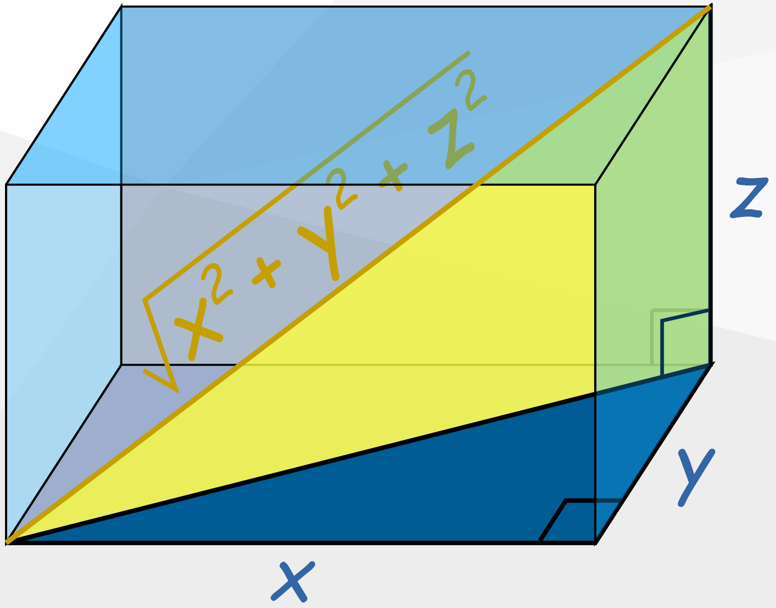
- What's next?
- What's the angle between the bottom face and the height (z)?

So, we can use Pythagoras's theorem in this 3rd dimension!

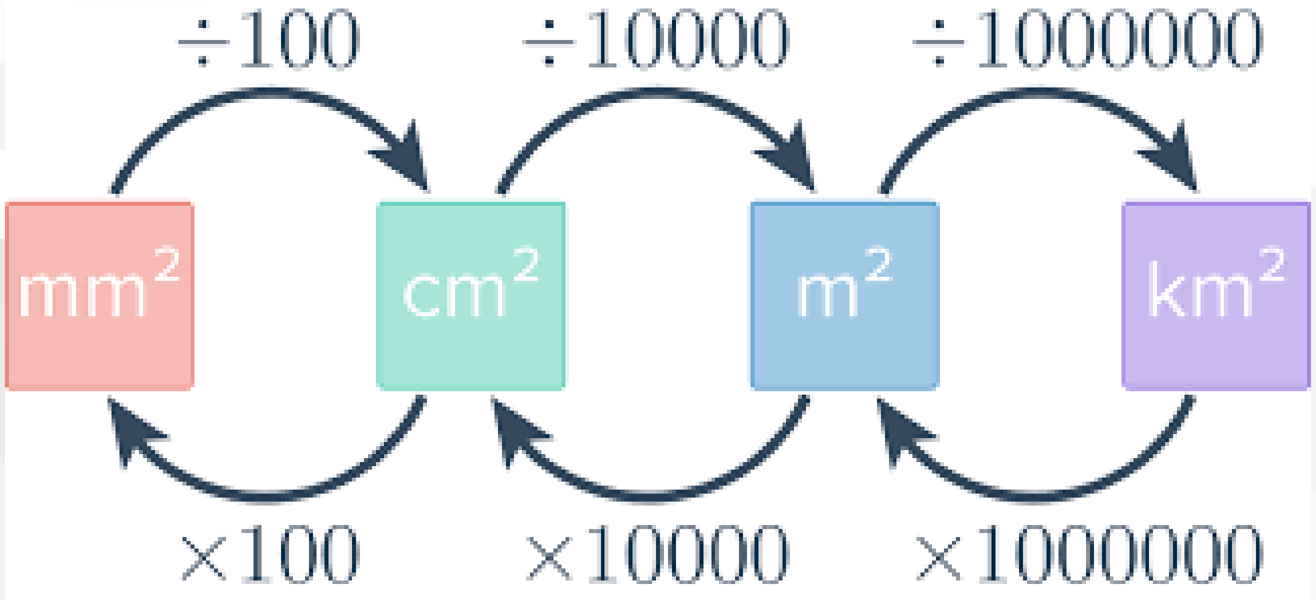


- What's our a and b?
- $a = \sqrt{x^2 + y^2}$ $b = z$
- $c^2 = a^2 + b^2$ (Pythagoras's Theorem)
  - $= (\sqrt{x^2 + y^2})^2 + z^2$

- $c^2 = (\sqrt{x^2 + y^2})^2 + z^2$
  - $= x^2 + y^2 + z^2$
- Taking the squareroot of both sides
- $\boxed{c = \sqrt{x^2 + y^2 + z^2}}$
- quite elegant!



Area: Unit Conversion

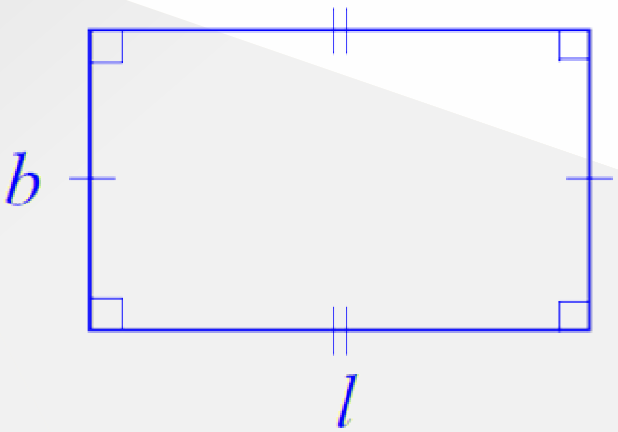


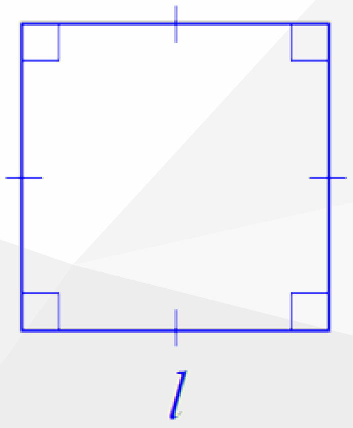
Think: Am I going from bigger to smaller?

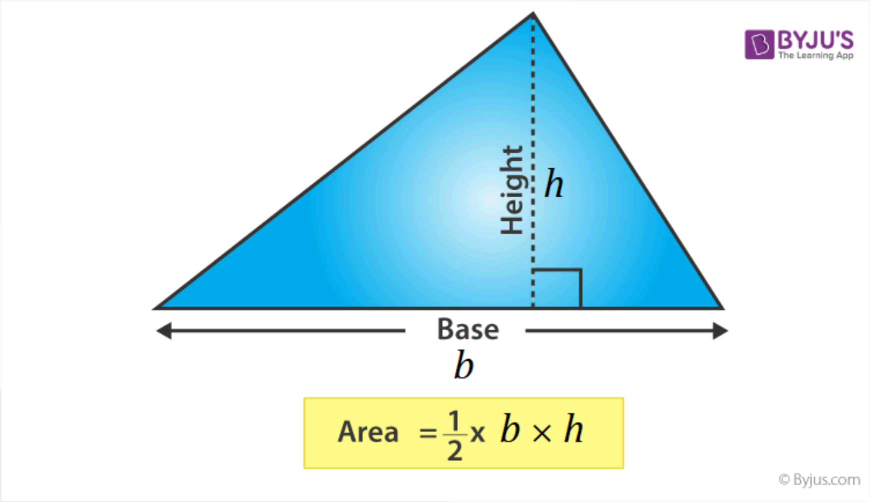
- Then there will be more units

Areas of 2D shapes

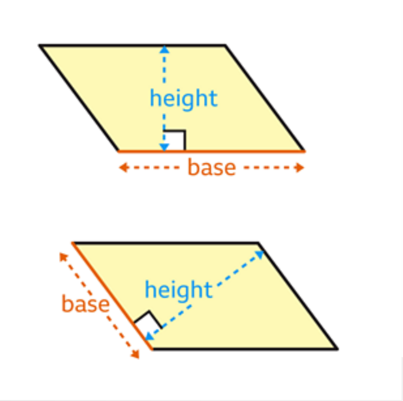
Rectangle	Square	Triangle
$A = l \times b$	$A = l^2$	$\frac{1}{2} b \times h$

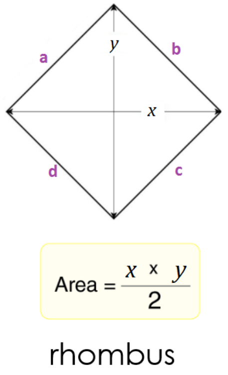


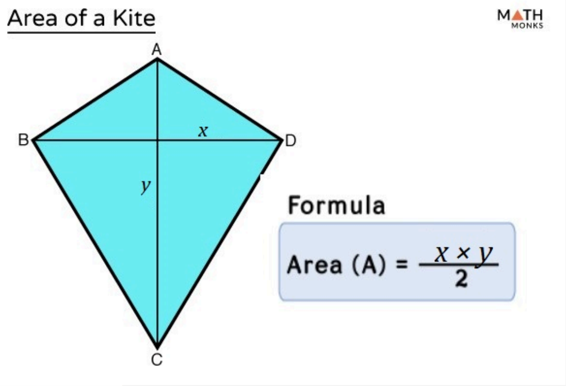


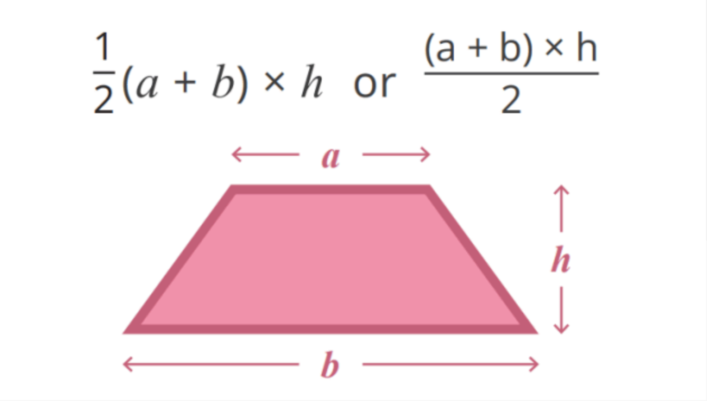


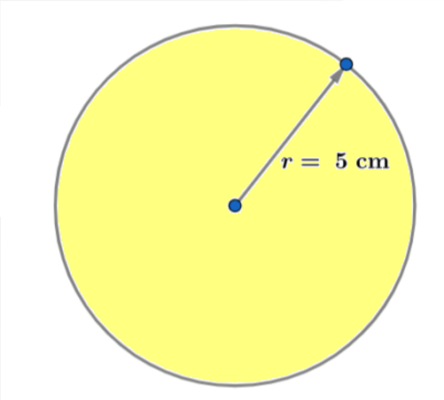
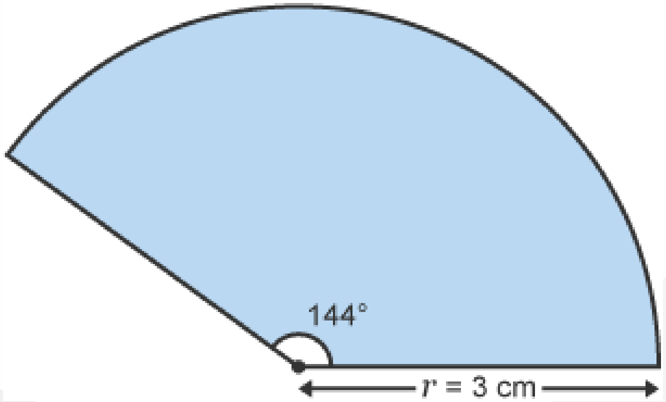
Parallelogram	Rhombus	Kite	Trapezium
$A = b \times h$	$A = \frac{x \times y}{2}$	$A = \frac{x \times y}{2}$	$A = \frac{a+b}{2} \times h$









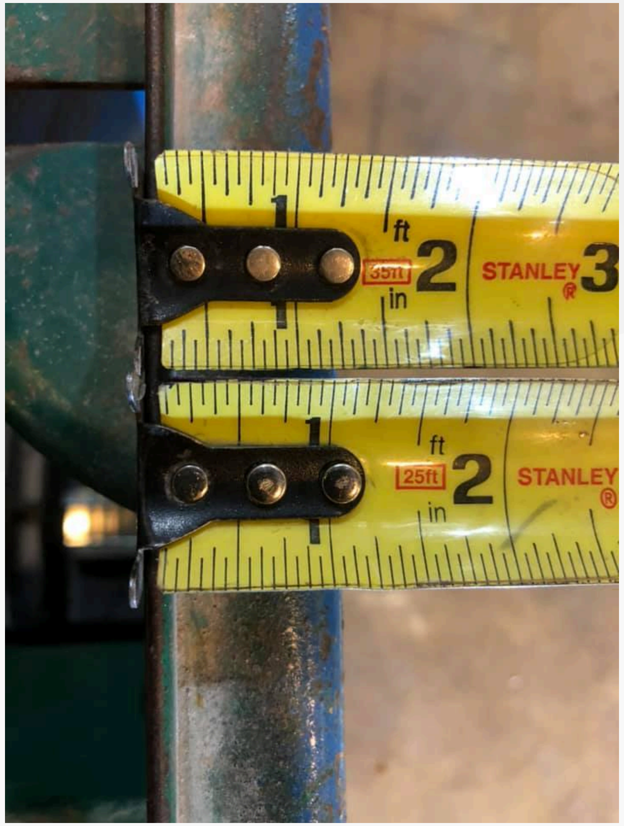
Circle	Sector
A = πr^2	A = $\pi r^2 \times \frac{\theta}{360^\circ}$
	

Accuracy

- In 1856, the Surveyor General of India Andrew Waugh measured Mt. Everest as exactly 29,000 ft
- But he announced it was 29,002 ft so people wouldn't think it was an estimate
- Historians called him "the first person to put two feet on top of Mount Everest"
- What does accuracy mean?
    - the measure of how close a recorded measurement is to the exact measurement

What could cause inaccurate readings?

- Faulty instruments
- Environmental factors: e.g. wind or uneven surface
- Procedural error, e.g. Parallax
- Human error, e.g. rounding or typos



A egg's length was recorded as 6.0cm, correct to the nearest millimetre.

a) What units were used when measuring?

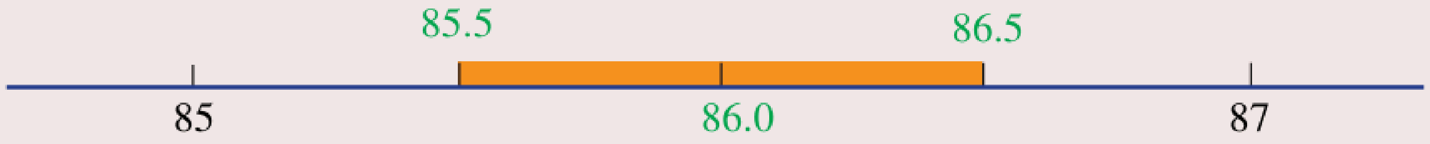
b) What is the largest decimal that would have resulted in 6.0cm?

c) What is the smallest decimal that could be rounded to this value?

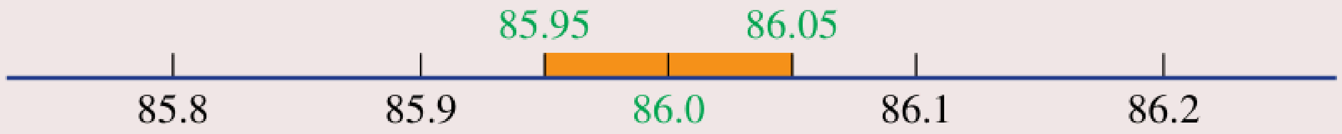
d) What mistakes could be made in measuring that would lead to an inaccurate reading?

KEY IDEAS

- The **limits of accuracy** tell you what the upper and lower boundaries are for the true measurement.
  - Usually, it is $\pm\ 0.5 \times$ the smallest unit of measurement.
    For example, when measuring to the nearest centimetre, 86 cm has limits from 85.5 cm up to (but not including) 86.5 cm.



  - When measuring to the nearest millimetre, the limits of accuracy for 86.0 cm are 85.95 cm to 86.05 cm.



- Errors can also occur in measurement calculations that involve a number of steps.
  - It is important to use exact values or a large number of decimal places throughout calculations to avoid an accumulated error.

Accumulated Errors

Complete these calculations.

a. i. 8.7×3.56, rounded to one decimal place

ii. Take your rounded answer from part a. i., multiply it by 1.8 and round to one decimal place.

b. i. 8.7×3.56 without rounding

ii. Take your exact answer from part b. i., multiply it by 1.8 and round to one decimal place.

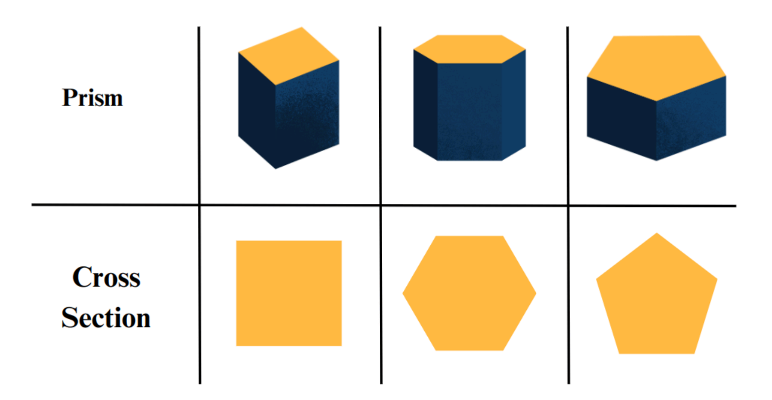
c. Compare your answers from parts a. ii. and b. ii. What do you notice? Which answer is more accurate?

Surface Area of Prisms and Cylinders

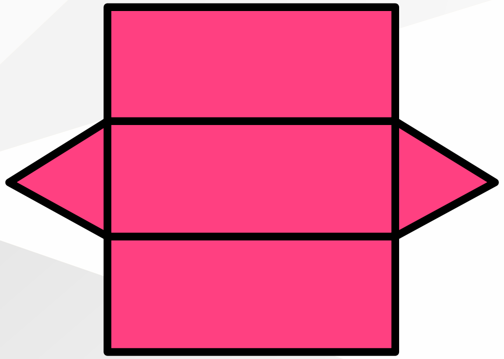
What is a **prism**?

- A solid 3d shape with a cross-section that is a **polygon**
  - cross-section: what you get when you slice a shape like bread

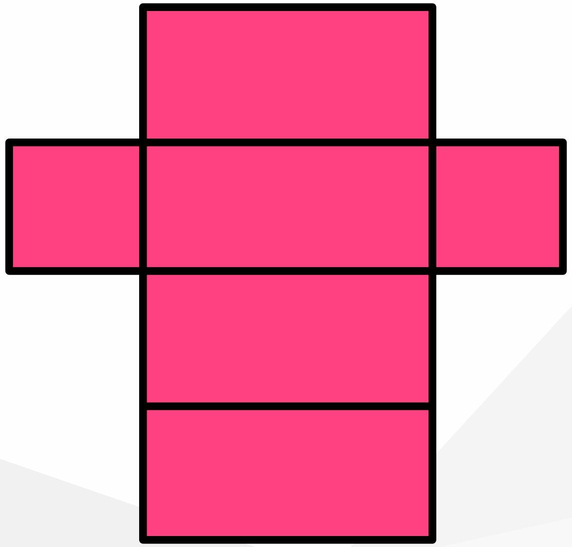
Prism: a solid with a **uniform polygonal cross-section**



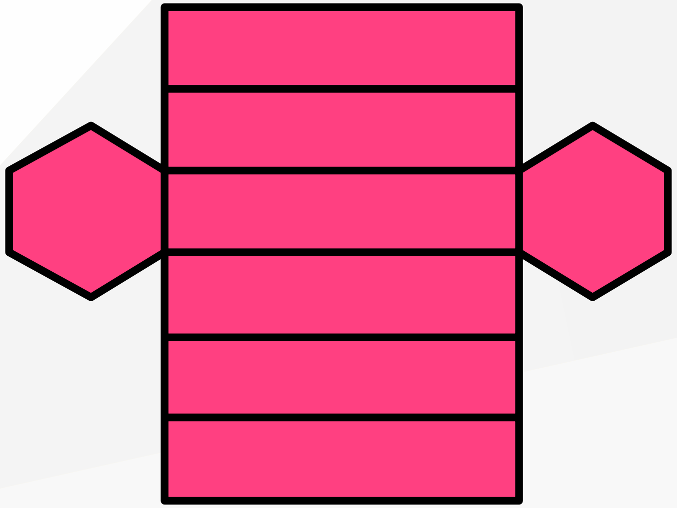
Finding the Surface Area of a Prism



Triangular Prism



Rectangular Prism (Cuboid)



Hexagonal Prism

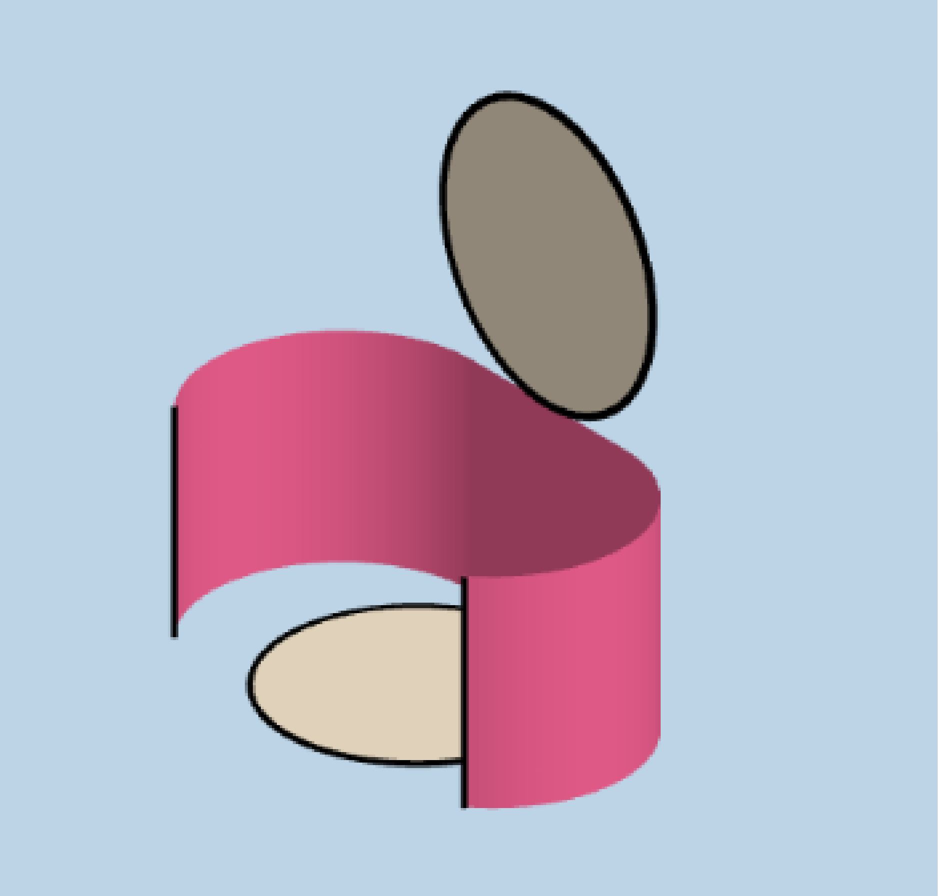
These are the **nets** of different prisms: the unfolded surfaces

To find the surface area, we find and add up the areas of each shape that makes up the prism's surface

What is a **cylinder**?

Like a circular 'prism': with a uniform circular cross-section

Surface Area



Formula

Area of the circles = πr^2 each

Area of the "tube" = $2\pi rh$ (Why?)

(We can unfold it into a rectangle with the circumference as a side)

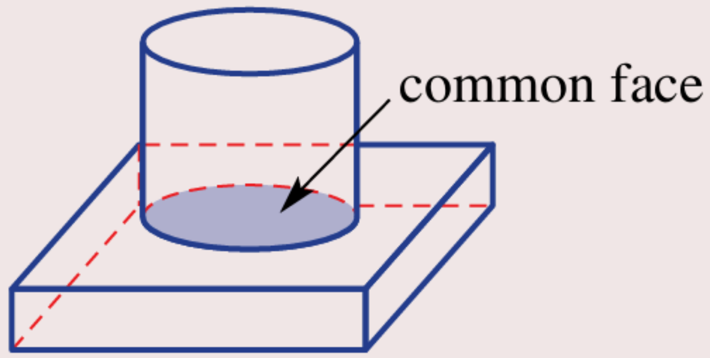
Put together, we get:

$$A = 2\pi r^2 + 2\pi rh$$

Composite Solids

Composite solids are solids made up of two or more basic solids.

- To find a surface area do not include any common faces.
  - In this example, the top circular face area of the cylinder is equal to the common face area, so the Surface area = surface area of prism + curved surface area of cylinder.

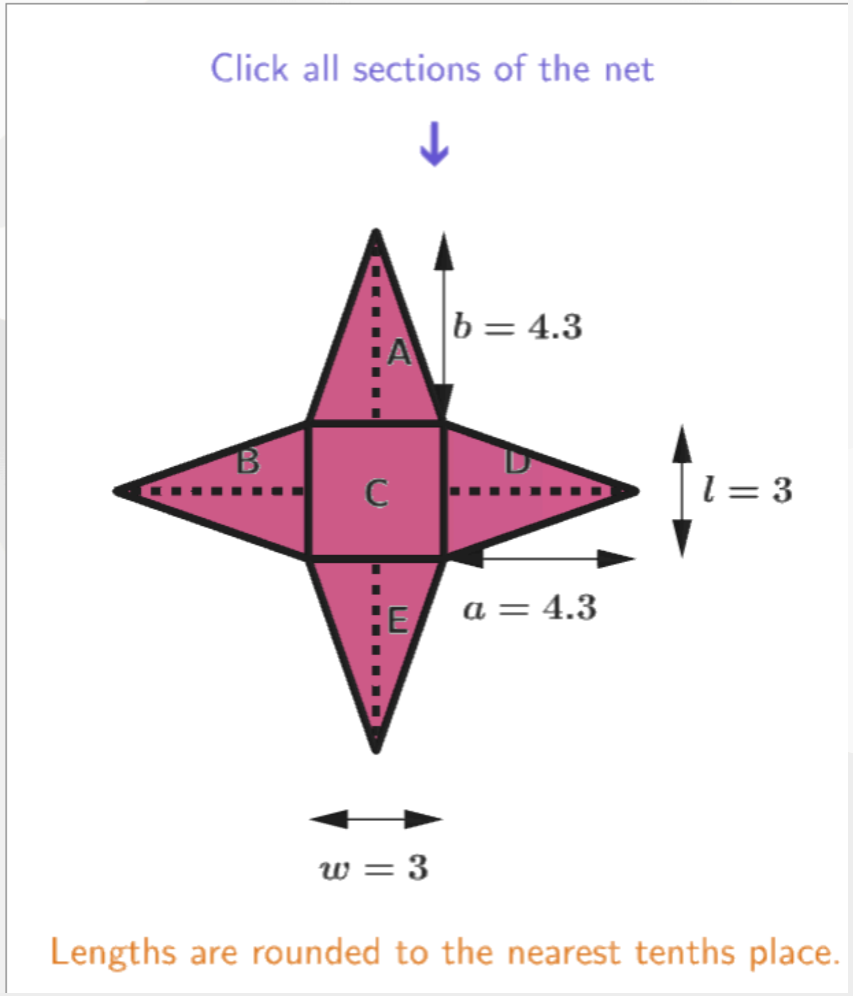


Surface Area: Pyramids and Cones

To find the area of a pyramid, we find

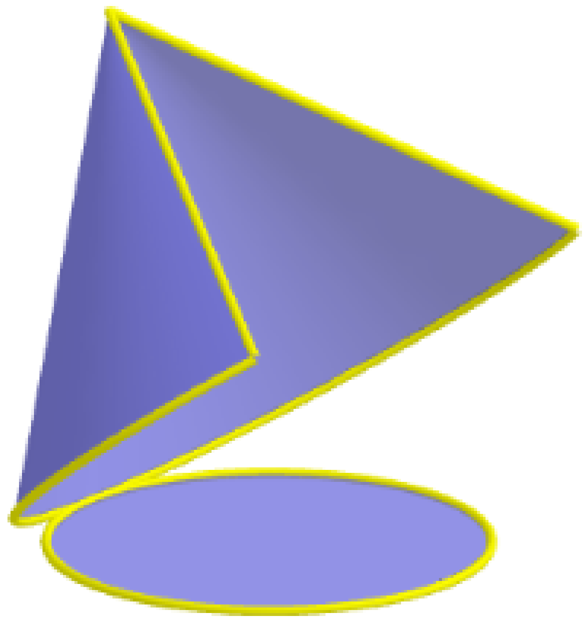
1. The area of the triangular sides
2. The area of the base

The number of triangular sides will depend on the type of pyramid.



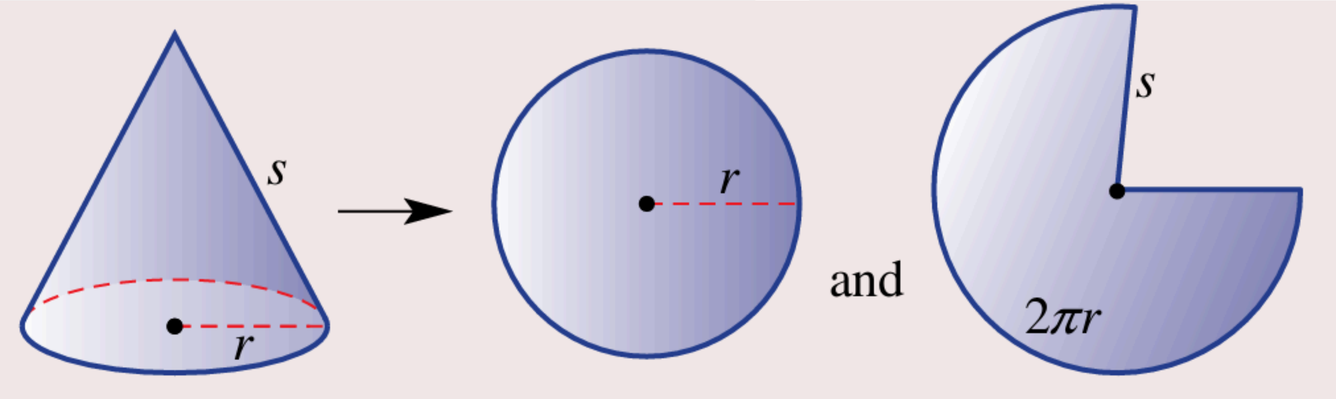
Cones

- a solid with a circular base
- a curved surface that reaches from base to apex (top point).
- A right cone has its apex directly above the centre of the base.
  - s = the slant height
  - r = radius of the base.



Surface Area

- The net of a cone is the circle at the base and a sector
- What's the arc of the sector?
  - The arc wraps around the circumference of the base circle
  - So the arc length is $2\pi r$
  - And the radius is s because we unwrap it from the slant



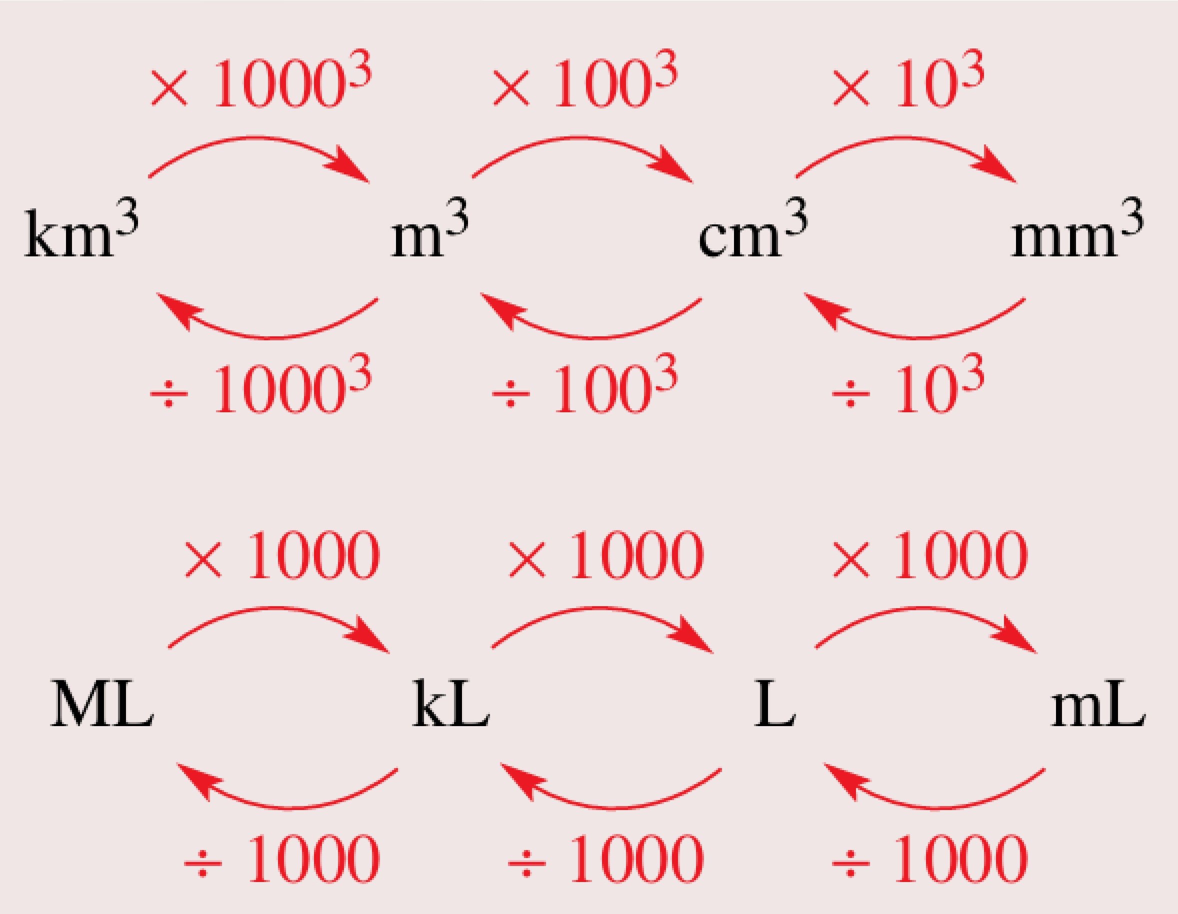
So what's the area of the sector?

- Area of the sector = $\pi s^2 \times \frac{\theta}{360^\circ}$
- But we don't know θ
- But! We do know the arc length
- Formula for arc of the sector = $2\pi s \times \frac{\theta}{360^\circ}$
- Actual arc = $2\pi r$
- So, $2\pi s \times \frac{\theta}{360^\circ} = 2\pi r$
- Cancelling, we get $s \times \frac{\theta}{360^\circ} = r$
- Dividing s from both sides, we get $\frac{\theta}{360^\circ} = \frac{r}{s}$
- So, our area is $\pi s^2 \times \frac{\theta}{360^\circ}$ = $\pi s^2 \times \frac{r}{s}$ = $\boxed{\pi r s}$

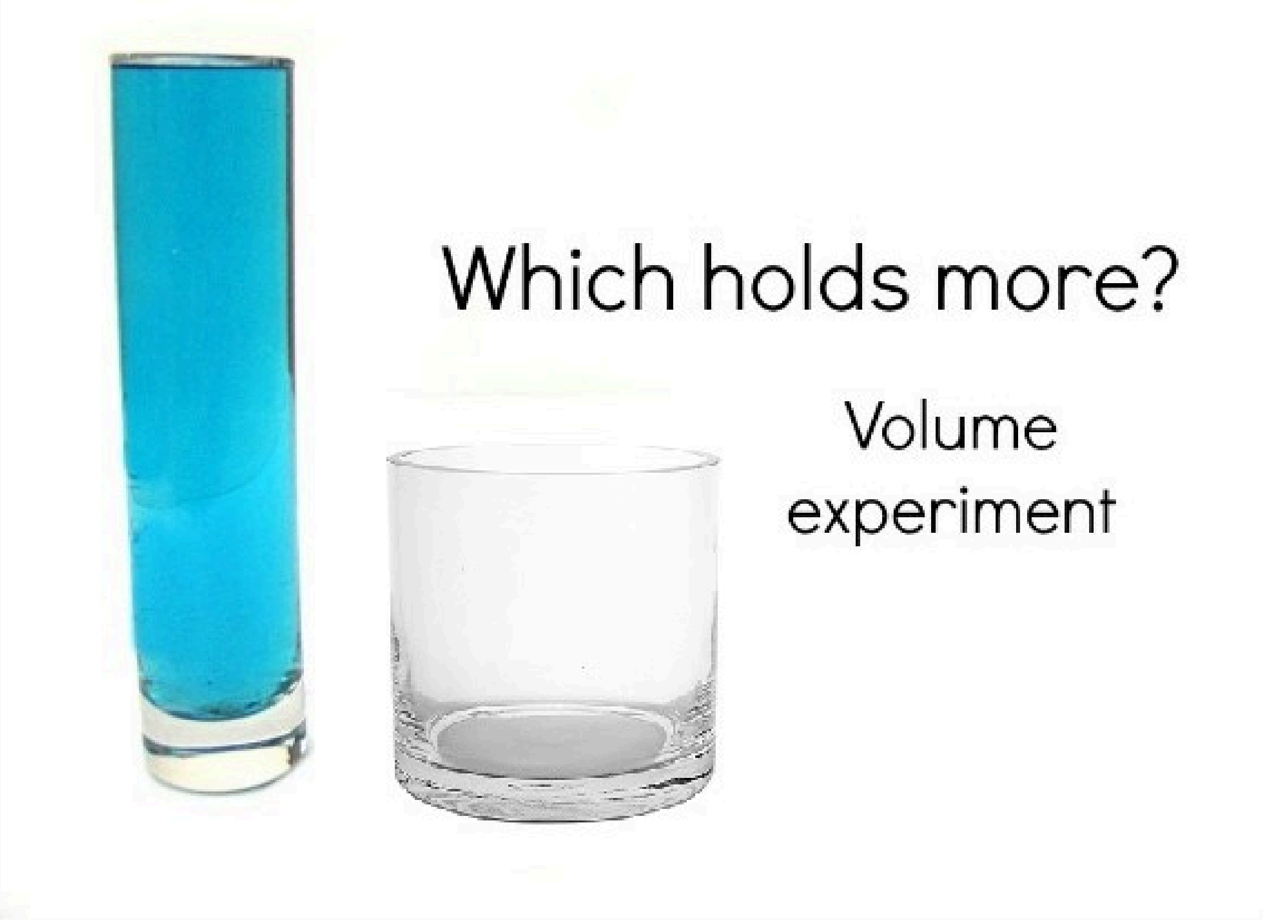
Now let's put that together with the base circle

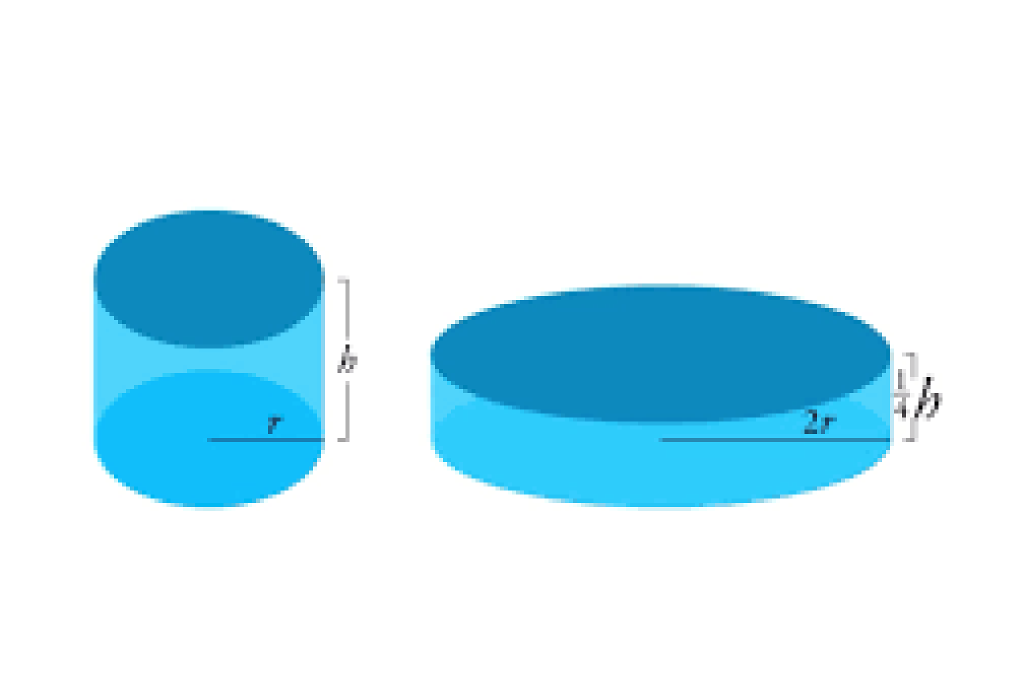
- Area of the base circle = πr^2
- Adding to the area we just found, we get
- $\boxed{\text{Surface Area of a Cone} = \pi r^2 + \pi r s = \pi r(r+s)}$

Volume and Capacity: Units



- Metric units for **volume**: km^3, m^3, cm^3, mm^3
- Units for **capacity**: megalitres (ML), kilolitres (kL), litres (L) and millilitres (mL).
  - $1\ cm^3 = 1\ mL$


Which holds more?
Volume
experiment


h
r
2r

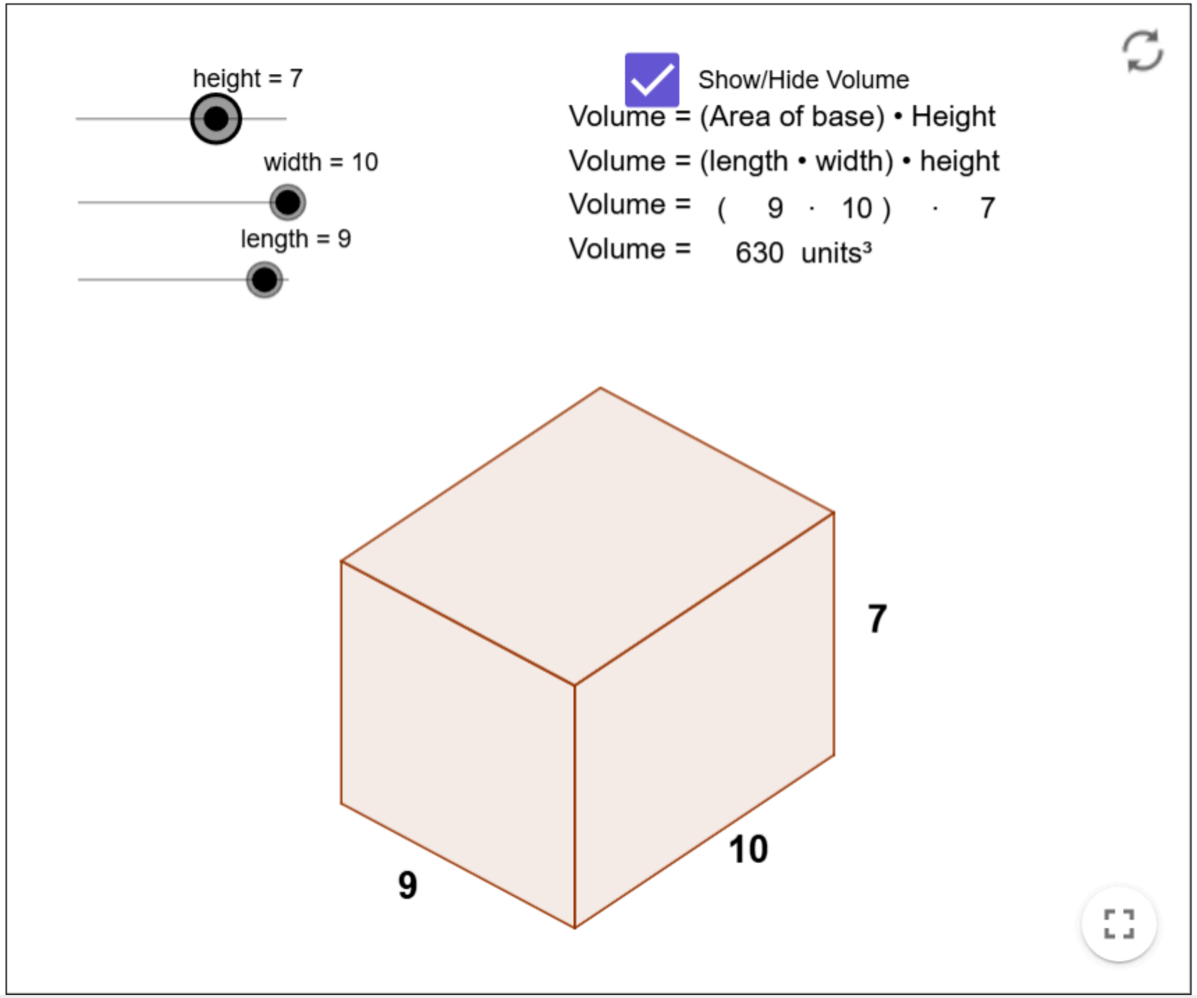
Volume: Prisms and Cylinders

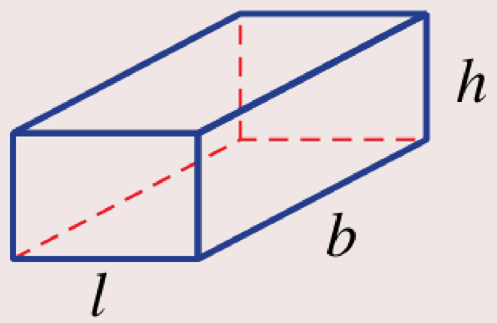
How does the volume change if the base or height changes?

- Volume = Area of base × Height

Formally:

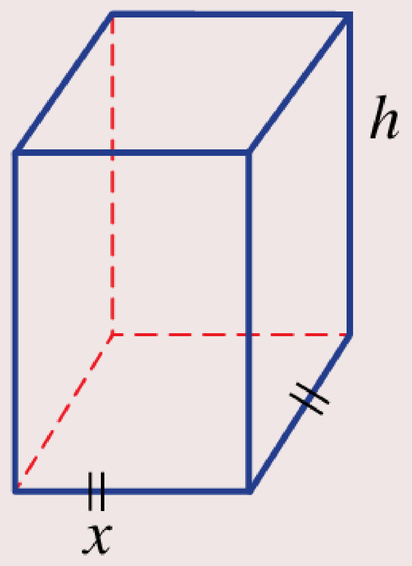
- For right prisms and cylinders, the volume $V = Ah$, where:
  - A is the area of the base
  - h is the perpendicular height.





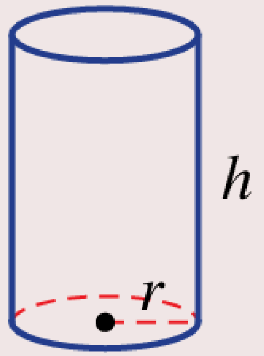
right rectangular prism

$$V = Ah$$
$$= lbh$$



right square prism

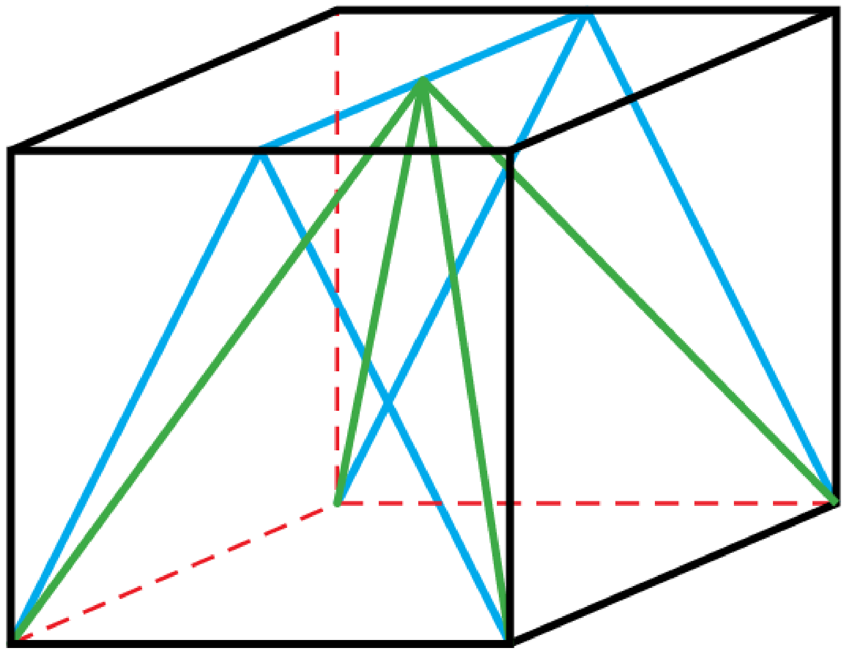
$$V = Ah$$
$$= x^2h$$



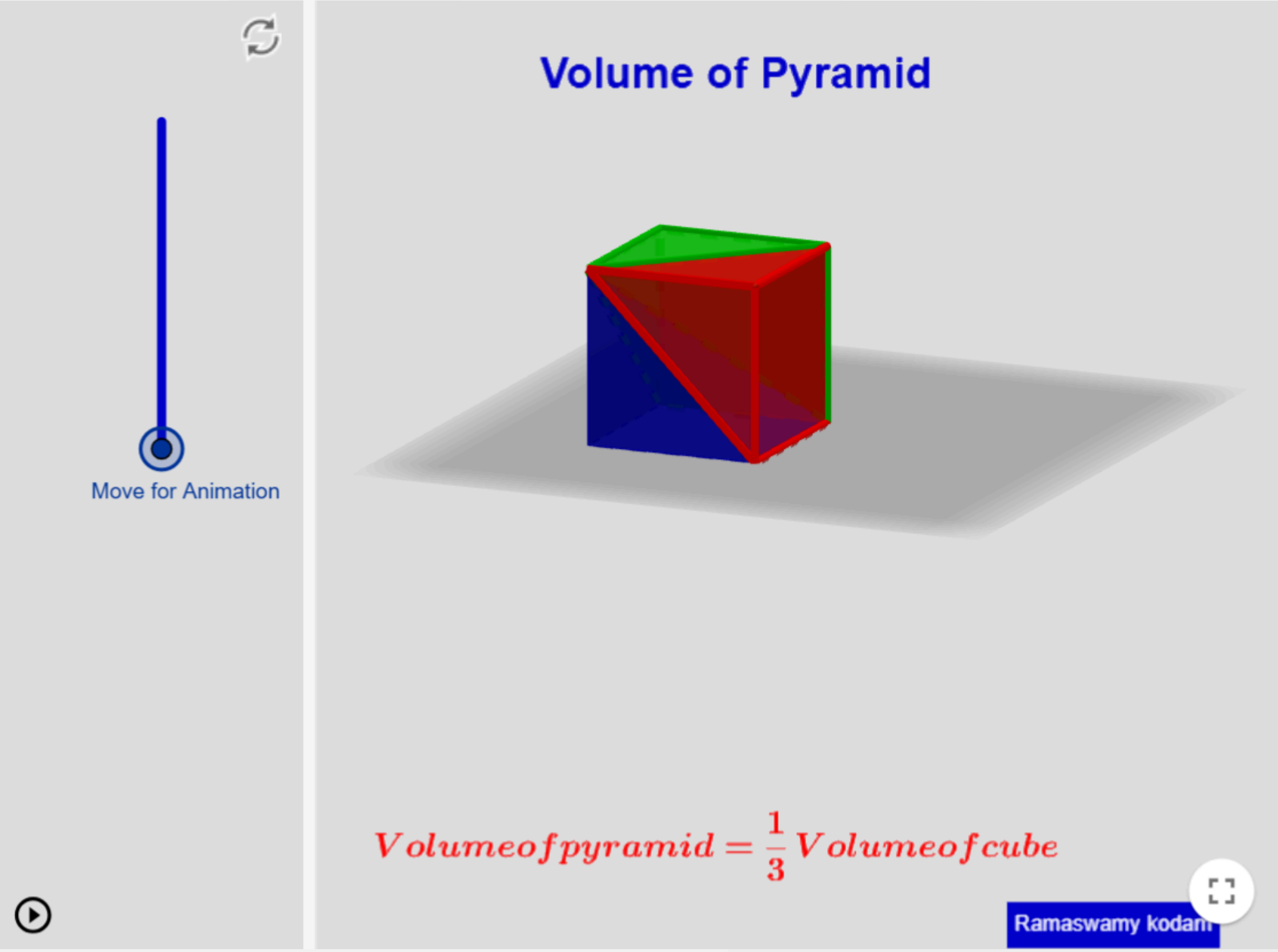
right cylinder

$$V = Ah$$
$$= \pi r^2 h$$

Volume: Pyramids and Cones



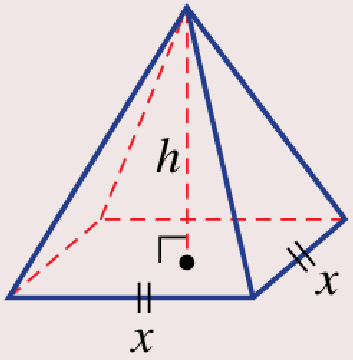
- What fraction are the triangular prism (blue) and pyramid (green) of the cube?
- How do we know?


Move for Animation
Volume of Pyramid
$Volume of pyramid = \frac{1}{3} Volume of cube$
Ramaswamy kodam

For pyramids and cones the volume is given by $V = \frac{1}{3}Ah$,

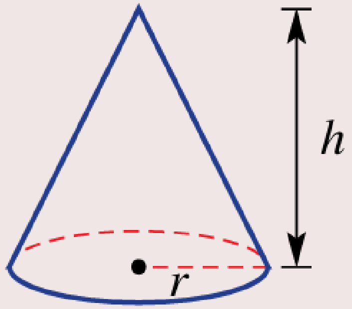
where A is the area of the base and h is the perpendicular height.

right square pyramid



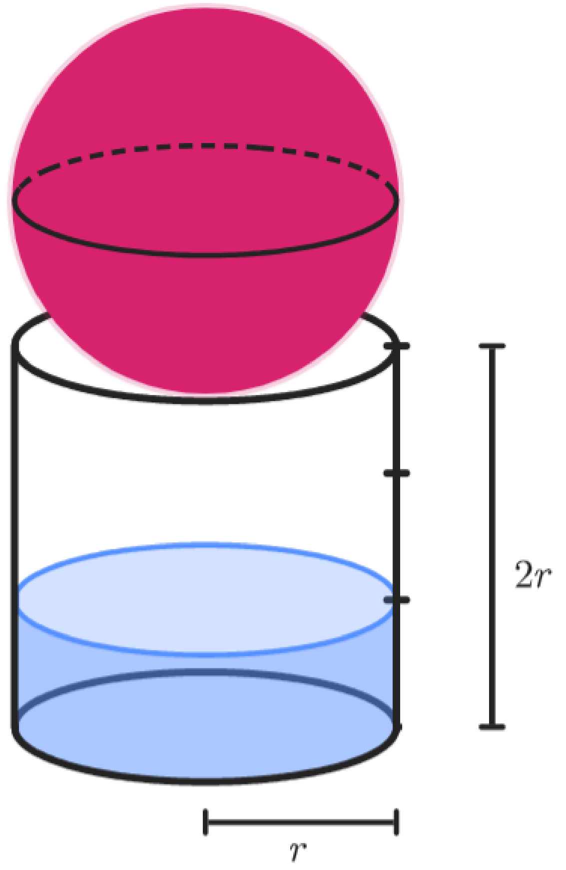
$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3}x^2h$$

right cone



$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3}\pi r^2h$$

Volume of Spheres



Pull the sphere out of the water.
Discover the formula for finding the volume of a sphere.

Volume of cylinder $= 2\pi r^3$

$$\begin{aligned} \text{Volume of sphere} &= \text{Volume of cylinder} \\ &\quad - \text{Volume of water} \\ &= 2\pi r^3 - \frac{1}{3}(2\pi r^3) \\ &= 2\pi r^3 - \frac{2}{3}\pi r^3 \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

ALTERNATE CALCULATION

The only dimension of a sphere is its radius r

The surface area is given by

$$\boxed{A = 4\pi r^2}$$

The volume is given by

$$\boxed{V = \frac{4}{3}\pi r^3}$$